Explaining a Simple Physical World
An introduction to *PDC*

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The following is a possible summary (`a draft written on 24/Apr/2006 and slightly corrected on 21/Apr/2009, and not meant for being a printable version`) of one of my major pieces of research. Any use, even partial, of original ideas contained in this material has to be referenced to the present paper.

As a provocative summon to criticism (and as a facilitation) I shall begin with the metaphysical tale of Psyche and Tyche.

Psyche and Tyche

Once upon a time there was a world named Hypoland. Hypoland was full of all kinds of both reasonable and bizarre hypotheses. In this world lived Psyche, a beautiful[2] mind supported by a very unlucky body[3], which could neither hear, touch, smell nor taste anything. Furthermore, Psyche's eyelids could not even shut. Psyche could only see and she was unfortunately obliged to perceive everything that was visible in this world.

This weird Hypoland was reigned over by Tyche[4][5], a playful Greek goddess, who delighted in[6] choosing the fate of the world by means of wheels of fortune.
Tyche had at her disposal a huge sack full of different wheels like the following[7] $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6$ of `Figure 1`:

By spinning a wheel every minute, Tyche could choose at random any tone of gray to color the whole world uniformly: the color pointed by the indicator when the wheel stops.

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[2]As in Greek mythology, Psyche was so beautiful as to make Eros (the god of love, also called Cupid) fall in love with her. They finally got married and had a daughter who was named Pleasure.
Psyche also became the personification of the soul, that is, of the mental substance.

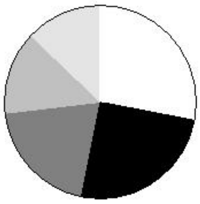
[3]This simple modification with respect to a brain-in-a-vat thought experiment should suffice to render ineffective all the counter-argumentations *à la* Putnam.

[4]Or otherwise known by the Roman counterpart Fortuna. In Greek, the word "tyche" means "chance".

[5]*Tych*ism is the name for the philosophical theory asserting that chance is an objective reality at work in the universe. Hypoland is clearly not objective though: if you can imagine that a certain dice has some chance, do you need to be an objectivist about chance?!

[6]In spite of Einstein's opinion (that God does not play dice).

[7]χ_1, χ_2, χ_3 stand as examples of discrete random variables, while χ_4, χ_5, χ_6 represent examples of degenerate, continuous and mixed-type random variables respectively.



Psyche has lived some time in this Hypoland, but eventually she gets bored of this apparent simplicity. So Psyche thinks she can be happier just if she manages to guess which color will appear next.

In order to achieve this, Psyche decides to indicate the color she sees by a lowercase Roman letter in italics: *a* for white[8], *b* for *b*lack, and so on.

Then Psyche starts taking rational notes of the colors she sees. Ideally Psyche wishes to manage to guess the next color by an analysis she makes on the past colors, concluding that some feature, like a proportion of a specific past color over all the past ones, will also hold for the future. So, she separates every letter representing a color by a comma and she indicates the division between a past color and a future one by a semi-colon: like $a, d, b, c; x$.

But, suddenly, Psyche questions her aim: 'How is it possible that a proportion of colors observed in the past has to remain the same also in the future? For what reason? Is that philosophically necessary?'

Then Psyche imagines there could be a goddess, such as Tyche, who is selecting the colors[9] by spinning a wheel.

'But, even if Tyche always spins the same wheel,' Psyche insists, 'why should it be possible or necessary that a proportion of colors observed in the past has to remain the same also in the future?'

Finally Psyche understands that this problem[10] is qualitatively equivalent to justifying why the most reasonable proportion of colors of a wheel which cannot be seen, but only "known" by the perceived colors, is the one of the experienced colors[11].

[8] Or *a*bsence of any black.
[9] These colors produced by wheels are Psyche's counterpart of realizations produced by random variables.
[10] The philosophical problem of induction.
[11] Therefore Psyche is for an *a priori* justification of induction.

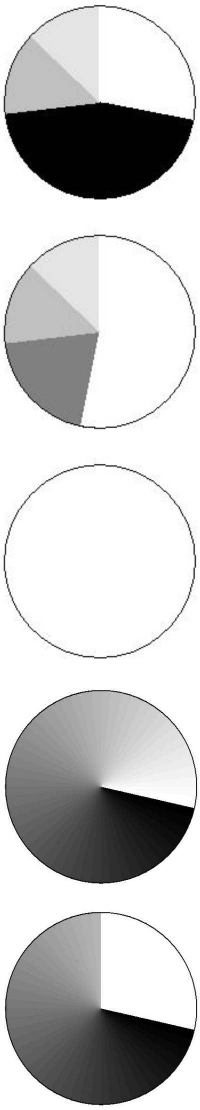


Figure 1

In fact, each spinning is independent from another one as well as every instant of time is independent from other ones, and any observed proportions need not correspond at all with the future proportion of colors. As the colors on the wheel are each seperately independent, so is past from future: the totality of the colors on the wheel stands for the totality of the possibilities in the time past plus future. 'But the time is infinite while the circumference of the wheel is finite!?!' 'Yes,' she continues, 'but then also on a finite wheel there can be infinite colors whereas over an infinite amount time there can be only a finite amount of colors.'

Then Psyche opts for approaching to the problem by distinguishing the past experimental colors from the future hypothetical ones, then by distinguishing the experimental proportions from the hypothetical proportions decided by Tyche in the future or in the past (but unknowable to Psyche directly), then by distinguishing all into *ex*perimental and *hy*pothetical.

So, in the hypothetical case that Tyche uses only the same wheel every time, how can Psyche infer the proportions of colors of the wheels that Tyche spins?

Psyche answers this question with her intuition: ‘By the hypothetical wheels that are most verified by the experience!’. In fact, the more a hypothetical wheel is verified with regard to the others, the “truer” Psyche would consider it.

Still, she has a problem: ‘But how to make up this system of verification and how to justify it?’

At this point, in order to dodge any further trouble, Psyche simplifies the problem even more by contemplating a wheel that has just a finite number of different colors, instead of an infinite one. She is determined in solving precisely at least the trivial case.

Then she has two fundamental observations, the first of which is the following:
‘If I want to verify the hypothetical wheel made of only one color[12] and I observe always that color, then I want that hypothetical wheel to be verified as the *t*otal number of observations. I will call this quantity as *transparency function* T. In fact, if for example the hypothetical wheel spinned by Tyche has a color in proportion equal to an irrational number, the experimental proportion (that is bound to be a rational number) will become more and more transparent to the real one as the number of observations increases.’

Later she thinks of the second observation:
‘There is still need of a further function that can discriminate between hypothetical wheels with the same transparency function with regard to the observed colors. I will call this second function as *umpire function* U for it acts as an official who judges disputes among contesting wheels. But how to define U?’

‘Well,’ Psyche adds, ‘given a certain series of observed colors and considering several hypothetical wheels, each of the latter ones has its specific different likelihood that is expressed by the well-known multinomial probability.’

[12]The degenerate case of χ_4.

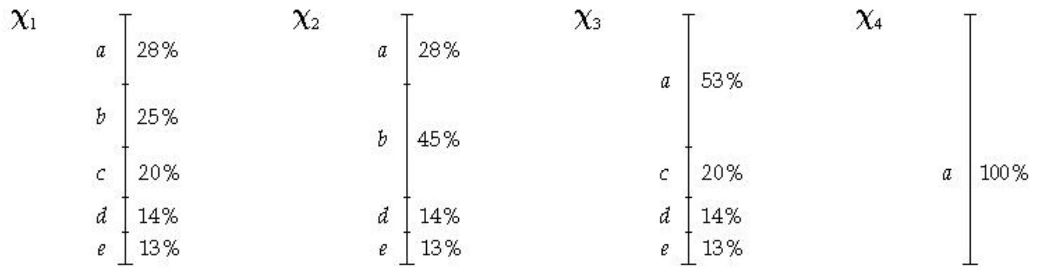
In fact, with this system of verification, it is easily provable that the most likely hypothetical wheel (meaning favorably-verified cases out of the total of verifiable cases) is the one that has the experimental proportions of colors.

At this point Psyche feels very happy about what she has realized and would like to obtain a working predictive system from it.

Nonetheless, Psyche's project still looks inachievable: in the general case Tyche can use different wheels whenever she likes and from only the observed colors Psyche aims to reconstruct the different wheels of the past and also the future ones!

How on Hypoland[13] might Psyche ever start to manage this?

According to the Hypoland's units of measurement, the circumference of a wheel of fortune is equal to one unit of length. Therefore, the arc of circumference of a specific color, which can be inferred from a wheel, numerically equals the probability of that color to occur.



`Figure 2`

After some time Psyche finds out some features of this world she is subject to. That is, she a-rationally feels that the durations of colors are integer multiples of her unit of time[14]. Then she also receives a rational confirmation.

So, Psyche starts to assign the letter of a color, which is hypothetically produced by a hypothetical wheel, to a discrete instant $1, 2, 3, \ldots$ like in the following[15] `Figure 3`:

She also chooses to indicate the *p*robability for the future events by a the lowercase

[13] The colors of Hypoland are intrinsically unpredictable *in principle*. However, this probabilistic nature – like the one ascribed by a standard interpretation of quantum mechanics – does not preclude the possibility of a strictly-deterministic nature, but it simply generalizes: the strictly-deterministic wheel χ_4 is just particular instances of "probabilistic" wheels.

[14] Psyche imagines in her view a hypothetical clock with a perfect motion of the second hand.

[15] However, Psyche cannot see what wheels Tyche chose in the past. She can only infer them from the experienced colors.

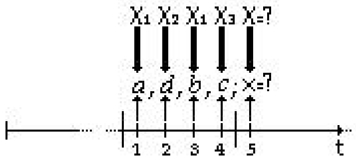


Figure 3

Greek letter Π and to call it *PDC*: *prescriptive degree of certainty.* In these terms, the problem is to determine Π_x.

Anyway Psyche can think about a lot of possible candidate wheels that can specify the observed *data*[16].

Here comes Psyche's third and final fundamental idea: 'If there isn't always the same wheel at every instant, then I call the assignation of some particular wheels to certain instants and the ways this is constructed as *CMS-theory* . The verification of a CMS-theory equals the sum of verifications of the parts with the same wheel and cause, by which the theory is constructed, unless one of these parts is not verified'.

Intuitively, a CMS-theory is a rational explanation of the behavior of data. And so, every hypothetical different theory is *w*eighted in a different way depending on how many times that theory is verified by the above system of verification.

From these considerations Psyche could make the best predictions for the next color and she could live happily ever after.[17]

Mathematical construction

On the ground of Psyche's words, it is easy and direct to construct a general theory which is, at the same time, a solution to the problem of induction, a theory of probability of data or theories, a theory of confirmation, a theory of causality and a theory of knowledge. The only illustrative case where data can be reduced to a string of data is treated here.

By definition, the last (discrete) instant of the past experience is indicated by n. Therefore $n + 1$ is the first (discrete) instant of the future.

[16] $a, b, c, d, e, \ldots$

[17] Psyche can also give an account of a world that is more general than the physical one if, besides the physical inputs x, she considers also the mental input y and the mental output z. This could be called the *CMS*-XYZ-world.

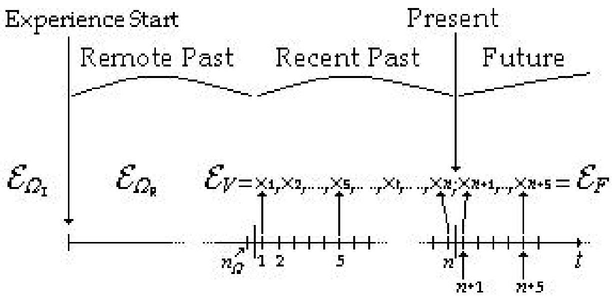


Figure 4

The generic datum x received at instant the generic instant t is indicated by x_t. Therefore, the series of data (string) which is taken into rational account[18] has the form of $\mathcal{E}_V = x_1, x_2, \ldots, x_{n-1}, x_n$. This period of rational activity is called *recent past* and $\mathcal{E}_V$ is called *rational past evidence.*

The period of time subsequent to the birth and prior the recent past is called *remote past* and the data a-rationally perceived in this period are indicated by[19] $\mathcal{E}_{\Omega_\mathrm{R}}$ which is called *remote evidence.*

Hypothetically existing innate information of data occurrence (already present at the instant of birth) is indicated by[20] $\mathcal{E}_{\Omega_\mathrm{I}}$ and called *innate evidence.*

The union of the innate evidence with the remote evidence $(\mathcal{E}_{\Omega_\mathrm{I}}, \mathcal{E}_{\Omega_\mathrm{R}} = \mathcal{E}_\Omega)$ is called *a-rational past evidence.*

The union of the a-rational past evidence with the rational past evidence $(\mathcal{E}_\Omega, \mathcal{E}_V = \mathcal{E}_P)$ is called *past evidence*[21].

A hypothetical evidence of future data is indicated as $\mathcal{E}_F$ and called *future evidence.*

With these definitions, the most general form of the problem of prediction is

$$\Pi\left(\mathcal{E}_P; \mathcal{E}_F\right) \tag{1}$$

but in this summary the only case of the *PDC* of a first future elementary datum x will be tackled $(\mathcal{E}_F = \mathbf{X} = X = x)$, that is to say

$$\Pi\left(\mathcal{E}_P; x\right) \tag{2}$$

[18]And thus, is subject to rational *v*erification.

[19]The remote past is as remote as Ω, that is, the last letter of the Greek alphabet.

[20]I stands for *i*nnateness.

[21]So, $\mathcal{E}_P = \mathcal{E}_{\Omega_\mathrm{I}}, \mathcal{E}_{\Omega_\mathrm{R}}, \mathcal{E}_V$.

that can be written in shorthand as

$$\Pi_x \tag{3}$$

An *CMS-rule* ρ, or rule *tout court*, is the ordered triple

$$\left(\mathcal{C}_\rho^{hy}, \lambda_\rho^{hy}, \chi_\rho^{hy}\right) \tag{4}$$

whereas ρ can take integral values and

$\mathcal{C}_\rho^{hy}$ specifies the *hypothetical cause of the rule* ρ;

λ_ρ^{hy} specifies the *hypothetical contextual location of the rule* ρ over the past and future time; and

χ_ρ^{hy} specifies the *hypothetical random variable*[22] *of the rule* ρ.

Informally speaking, a rule assigns one specific wheel χ_ρ^{hy} to a set of instants λ_ρ^{hy} by means of a certain hypothetical cause $\mathcal{C}_\rho^{hy}$.

The hypothetical contextual location of the rule ρ will then be a subset of the integer numbers:

$$\lambda_\rho^{hy} = \{t\} \subseteq \mathbb{N} \tag{5}$$

it contains T_ρ numbers, i.e., the rule ρ is of T_ρ instants (T_ρ can be finite or infinite) and it also fulfills the condition that it must *not* be specified by a piecewise function[23].

By contrast, the *experimental contextual location of the rule*

$$\lambda_\rho^{ex} = \left\{t \leq n \,\text{with}\, t \in \lambda_\rho^{hy}\right\} \tag{6}$$

indicates the instants of the rule ρ which already belong to the past. T_ρ^n is the number of past instants[24] of the rule ρ. The *substring of* $\mathcal{E}_P$ *given by the past data of the rule* ρ is

$$\mathcal{E}_{\lambda_\rho^{ex}} = \left\{x_t \in \mathcal{E}_P \,\text{with}\, t \in \lambda_\rho^{ex}\right\} \tag{7}$$

The data that a random variable make them to be actually observed are also called *realizations* of the random variable.

[22] χ_ρ^{hy} serves to represent the *ch*ance. Note: here an X would stand for a string.

[23] Justifications of this conditions are clear thinking about an *a*pple that freezes in time (see below). Anyway this point is very interesting and its development is in support of (a pragmatic) constructivism.

[24] The last experiment*ed* and verifiable instant of the rule is $n = \max\left\{\lambda_\rho^{ex}\right\}$.

The *sample space* σ is the set of all hypothetically possible realizations. Thus it always contains infinite elements and, obviously, also all the realizations of the past.

The hypothetical cause of the rule is essentially a property defining λ_ρ^{hy} and χ_ρ^{hy}.

The type of hypothetical cause determines the type of rule, that can be:

off line if λ_ρ^{hy} is exactly determinable prior to having got the particular realizations of the experience; or

on line if otherwise.

There might be the rule with $\rho = 1$ meaning that Tyche illuminates with white (chooses the wheel χ_4) after each realization of black (and it may occur, for instance, that this is generated by the wheel χ_1, but this is not knowable before χ_1 is specified), in which case ρ is an on-line rule.

There might be also the rule with $\rho = 4$ that Tyche illuminates with white at every instant t that is an integer multiple of 4 (4,8,12,...), in which case ρ is an off-line rule.

But there might also be the rule with $\rho = 16$ that Tyche, after instant 17, decides to illuminate with white at instant 18 according to no general property, in which case ρ is still an on-line rule (because it is not possible to determine exactly λ_ρ^{hy} prior to any realization. Clearly, an on-line treatment is more general and exact than an off-line one.

Two different hypothetical causes can apply to the same phenomenology: for example, the tides can have, as a hypothetical cause 1, the position of the moon, or, as a hypothetical cause 2, the gravitation law (although this second law applies to a wider phenomenology). On the other hand the concept of hypothetical cause is quite flexible: for example, the hypothetical cause of a rule could be seen as a set of more elementary hypothetical causes.

The set of the considered CMS-rules is indicated by $\mathbf{R} = \{\rho \text{ with } \rho = 1, 2, \ldots, N_\rho\}$.

On the instants given by λ_ρ^{ex} there is defined the *experimental probability of datum x according to the rule ρ* as

$$\Pi_{x\rho}^{ex} = \frac{F_{x\rho}^n}{T_\rho^n} \tag{8}$$

in other words, it is the proportion at $t = n$ of $F_{x\rho}^n$ (experimentally and conclusively) *f*avorably-*verified* cases for datum x in the set of T_ρ^n *t*otal of (experimentally and conclusively) *verifiable* cases of the rule ρ (for $t \leq n$).

What follows is that

$$\sum_{x\in\sigma} F^n_{x\rho} = T^n_\rho \tag{9}$$

and, as T^n_ρ does not depend on a particular datum, the following relation is implied

$$\sum_{x\in\sigma} \Pi^{ex}_{x\rho} = 1 \tag{10}$$

Indicated by $\Pi^{hy}_{x\rho}$ the *hypothetical probability of datum x (at instant $t = n+1$) according to the rule ρ*, the definition of wheel yields[25]

$$\sum_{x\in\sigma} \Pi^{hy}_{x\rho} = 1 \tag{11}$$

Despite the similarity in the names, it is useful to note crucial differences: $\Pi^{ex}_{x\rho}$ applies only to the past, Π_x only to the future, while $\Pi^{hy}_{x\rho}$ can refers to both of them. Besides, $\Pi^{ex}_x \in \mathbb{Q}$ while $\Pi^{hy}_x \in \mathbb{R}$[26].

It is now possible to talk about *rational verifications of a rule at instant n* (n included) V^n_ρ: this is achieved by the product of the *t*ransparency function by the *u*mpire function:

$$V^n_\rho = T^n_\rho \cdot U^n_\rho \tag{12}$$

V is called *prescriptive rational verification function*, for it applies to the only period of rational analysis: the recent past. The verifications V of the rule are obviously meant as a *non-conclusive* verification.

Thanks to Psyche, it is now clear that the transparency function is actually given by the denominator of the (8) and the umpire function is simply the multinomial probability of a hypothetical wheel χ^{hy}_ρ (of the rule ρ) given the evidence $\mathcal{E}_{\lambda^{ex}_\rho}$.

The multinomial probability quantifies the likelihood of obtaining exactly, out of n realizations from the wheel χ^{hy}_ρ, $F^n_{a\rho}$ favorably-verified cases for datum a, $F^n_{b\rho}$ favorably-verified cases for datum b, $F^n_{c\rho}$ favorably-verified cases for datum c, and so forth[27]. It can be expressed by the following

$$U^n_\rho = \eta^n_\rho \cdot \theta^n_\rho \tag{13}$$

[25] $\Pi^{hy}_{x\rho} \in \rho^{hy}_x$. Also a χ^{ex}_x can be made up from the $\Pi^{ex}_{x\rho}$'s.

[26] Other possible analogous definitions are $F^{hy}_{x\rho} = \Pi^{hy}_{x\rho} \cdot T_\rho$ and $F^{hyex}_{x\rho} = \Pi^{hy}_{x\rho} \cdot T^n_\rho$ which are the hypothetical absolute real number of favorably-verified cases generated with a relative frequency exactly equal to the hypothetical probability over all instants of ρ and over all experiment*ed* instants of ρ respectively.

[27] And this independently from their order as each spinning is independent from the others.

θ^n_ρ is the probability of getting the string $\mathcal{E}_{\lambda^{ex}_\rho}$ in its specific order by subsequent realizations of χ^{hy}_ρ. The occurrence of a realization x has the probability $\Pi^{hy}_{x\rho}$ and, for the probability of the arrangement $\mathcal{E}_{\lambda^{ex}_\rho}$, all hypothetical probabilities of every occurred realization are multiplied together by virtue of the assumption of independent spinnings:

$$\theta^n_\rho = \left(\Pi^{hy}_{a\rho}\right)^{F^n_{a\rho}} \cdot \left(\Pi^{hy}_{b\rho}\right)^{F^n_{b\rho}} \cdot \left(\Pi^{hy}_{c\rho}\right)^{F^n_{c\rho}} \cdot \ldots = \prod_{x\in\sigma} \left(\Pi^{hy}_{x\rho}\right)^{F^n_{x\rho}} \tag{14}$$

and it can also be written as[28]

$$\theta^n_\rho = \prod_{x\in\sigma} \left(\Pi^{hy}_{x\rho}\right)^{\Pi^{ex}_{x\rho} \cdot T^n_\rho} \tag{15}$$

But since this probability applies to any sequence of n data in which there are $F^n_{a\rho}$ a's, $F^n_{b\rho}$ b's, ..., and since the realizations of the spinnings are not dependent on the order, to get the corresponding probability U^n_ρ of those realizations in any order Psyche has just to count the number of sequences of that kind that there are and then multiply this number η^n_ρ by θ^n_ρ. η^n_ρ is simply the multinomial coefficient given by[29]

$$\eta^n_\rho = \binom{T^n_\rho}{F^n_{a\rho}, F^n_{b\rho}, \ldots} = \frac{T^n_\rho!}{\prod_{x\in\sigma} \left(\Pi^{ex}_{x\rho} \cdot T^n_\rho\right)!} \tag{17}$$

and it can also be written as

$$\eta^n_\rho = \binom{F^n_{a\rho} + F^n_{b\rho} + F^n_{c\rho} + \ldots}{F^n_{a\rho}, F^n_{b\rho}, F^n_{c\rho}, \ldots} = \frac{\left(\sum_{x\in\sigma} F^n_{x\rho}\right)!}{\prod_{x\in\sigma} \left(F^n_{x\rho}!\right)} \tag{18}$$

Finally, a rule ρ can be said to have been verified V^n_ρ times, and this number can be real since U^n_ρ is a real number. Need to say also that V^n_ρ is maximum when $\forall x \in \sigma\ \Pi^{hy}_{x\rho} = \Pi^{ex}_{x\rho}$.

A *CMS-theory* $\mathcal{T}_i$, (or *theory tout court*), is a set of CMS-rules whose λ^{hy}_ρ's are pairwise disjointed, that is, every rule of a theory is located in different positions. A theory can

[28] It is also natural to define as the *rational falsifications of a rule at instant n* (n included) $\digamma^n_\rho$ the sum of the (non-zero) exponents $F^n_{x\rho}$ of the zeros of θ^n_ρ at instant n (n included).

[29] Obviously, $n!$ is the factorial of $n \in \mathbb{N}$ defined as

$$n! = \begin{cases} \prod_{i=1}^{n} i & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases} \tag{16}$$

However, in the most general case that deals thoroughly also with vagueness, the factorials of F will have to be replaced by the Gamma function $\Gamma(\ F+1)$: $\eta^n_\rho = \frac{\Gamma\left(1+\sum_{x\in\sigma} F^n_{x\rho}\right)}{\prod_{x\in\sigma} \Gamma\left(1+F^n_{x\rho}\right)}$.

contemplate different wheels in its contextual location[30], yet, only one at each instant.

A *CMS-theory* $\mathcal{T}_i$, is a *model* $\mathbf{m}_i$ if it does no mention of any cause at all[31].

A theory $\mathcal{T}_i$ is called an *omnipresent theory* if $\lambda_{\mathcal{T}_i}^{hy} = \mathbb{N}$, that is, the rules of the theory cover every instant of the time line from the start of the rational analysis.

The set of all the considered CMS-theories is indicated by $\mathbf{T} = \{\mathcal{T}_i \text{ with } i = 1, \ldots, N_{\mathcal{T}}\}$

A rule ρ is called a *relevant rule with probability* $\Pi_{x\rho}^{hy}$ if $t = n + 1 \in \lambda_\rho^{hy}$.

The relevant rule is "relevant" with respect to the calculation of Π_x. The just-mentioned $\Pi_{x\rho}^{hy} \in \chi_\rho^{hy}$ is the hypothetical probability of the datum x at the instant $t = n+1$ according to ρ. Consequentially, $t = n + 1$ is requested to belong to λ_ρ^{hy} by definition of relevant rule, but the same cannot belong to λ_ρ^{ex}), by definition of n. The set of all relevant rules is indicated by $\mathscr{R}$. $\mathscr{R} \subseteq \mathbf{R}$ holds.

A theory $\mathcal{T}_i$ is called a *relevant theory with probability* $\Pi_{x\tau_i}^{hy}$ and indicated by τ_i if it contains a relevant rule with probability $\Pi_{x\rho}^{hy}$ (in which case $\Pi_{x\tau_i}^{hy}$ is equal to $\Pi_{x\rho}^{hy}$ by definition). The index i of a relevant theory τ_i is the same of the corresponding theory $\mathcal{T}_i$. The set of all relevant theories of the same value of probability $\Pi_{x\tau_i}^{hy}$ is indicated by $\mathscr{T}$ and its elements will depend on the considered probability $\Pi_{x\tau_i}^{hy}$. Clearly $\mathscr{T} \subseteq \mathbf{T}$ holds.

$\Pi_{x\tau_i}^{hy}$ is called the *hypothetical probability of datum x (at instant $t = n + 1$) according to the theory τ_i*, and, as it always refers to the hypothetical probability according to a relevant rule, the *a priori* definition of wheel leads to

$$\sum_{x \in \sigma} \Pi_{x\tau_i}^{hy} = 1 \tag{19}$$

A theory, a model, or a rule are said to be *hypothetical* for they refer to hypothetical causes, contextual locations and random variables.

Exploiting Psyche's last fundamental observation, it is now possible to define the *rational verifications of a relevant theory at instant n* (n included) and this is achieved by

[30] It is natural to define $\lambda_{\mathcal{T}_i}^{hy} = \bigcup_{\rho \in \mathcal{T}_i} \lambda_\rho^{hy}$ and $\lambda_{\mathcal{T}_i}^{ex} = \bigcup_{\rho \in \mathcal{T}_i} \lambda_\rho^{ex}$. The last verifiable instant of the theory $\mathcal{T}_i$ is $N_{hy} = \max\left\{\lambda_\rho^{hy}\right\}$.

[31] According to Katherine Brading, there is the following essential difference between models and theories: 'A theory is expressed linguistically and it specifies a collection of models. A model is a collection of objects that makes the sentences of our theory true'. In my account similarly, models are the part of events in some location by some chance, while theories offer also a part on the causes, which is related to a non-mathematical language in general.

introducing the *o-plus operator*[32]:

$$V^n_{\tau_i} = V^n_{\tau_i}\left(\Pi^{hy}_{x\tau_i}\right) = \bigoplus_{\rho\in\tau_i} V^n_\rho = \begin{cases} \sum\limits_{\rho\in\tau_i} V^n_\rho \quad \text{if} \quad \prod\limits_{\rho\in\tau_i} V^n_\rho > 0 \\ 0 \quad \text{if} \quad \prod\limits_{\rho\in\tau_i} V^n_\rho = 0 \end{cases} \tag{20}$$

This operator makes the *rational verifications of a relevant theory* $V^n_{\tau_i}$ either equal to the sum of its rules or zero when at least one $V^n_\rho = 0$. This happens either when there is a falsified datum ($U^n_\rho = 0$) or when a rule of the theory is not verifiable yet ($T^n_\rho = 0$).

As the PV depends on only the $F^n_{x\rho}$ of every datum x (that belongs to the sample space) then V does not depend on the choice of a particular datum x (of a hypothetical random variable).

The case where all data are not available to a rational reckoning, such as data from innateness or remote past, has not been treated yet. It is for this purpose that the *a-rational verification function* Ω is introduced[33]. $\Omega^{t_\Omega}_\rho$ serves also as an useful tool for the integration of background knowledge.

$\Omega^{t_\Omega}_\rho$ the *a-rational verifications of a rule at instant* t_Ω (t_Ω included), and all other corresponding a-rational quantities, ought to be assessed subjectively in a way such as if data of the a-rational data were ideally be placed in the recent past: $\Omega^{n_\Omega}_\rho$ ought to be assessed in the same way that the V functions ideally. Rational tools for aiding in a good assessment of the "a-rational computation" of the Ω functions can be provided: for instance, the fact that any a-rational verification function must be the same for every datum of theory. There are also some other rational aids, like the fact that the sum of hypothetical or experimental probabilities of every datum makes one (therefore representable on an "experimental" wheel), can be helpful for a "good rationalization" of the a-rational information of verifications. Another one is the following: very often we own a psycholog-

[32] The name is suitable: this operator is either the sum (plus) or zero (o).

It is also useful to define $T^n_{\tau_i} = \bigoplus\limits_{\rho\in\tau_i} T^n_\rho$ and $U^n_{\tau_i} = \bigotimes\limits_{\rho\in\tau_i} U^n_\rho = \begin{cases} \prod\limits_{\rho\in\tau_i} U^n_\rho \quad \text{if} \quad \prod\limits_{\rho\in\tau_i} U^n_\rho > 0 \\ 0 \quad \text{if} \quad \prod\limits_{\rho\in\tau_i} U^n_\rho = 0 \end{cases}$.

[33] According to the Russellian "ten-minute hypothesis" we could have appeared in this universe just 10 minutes ago. Yet what we have as evidence remains evident, as a psychological sensation. At most, the *theories* "governing" our future evidence can have nothing to do with what it seems reasonable to infer on the basis of the evidence that we have. ..But this is another story. Anyway Ω offers a framework also for these problems.

ical feeling of the Bayesian priors $\mathring{\Pi}^{n\Omega}_{\tau_i}$'s much clearer than $\Omega^{n\Omega}_{\tau_i}$; but, if one $\Omega^{n\Omega}_{\tau_i}$ can be estimated accurately, for instance the maximum of the $\Omega^{n\Omega}_{\tau_j}$'s of the relevant theories that must be equal to $\frac{\mathring{\Pi}^{n\Omega}_{\tau_j}}{\mathrm{C}}$ (where C is a constant, thereby $\frac{\mathring{\Pi}^{n\Omega}_{\tau_j}}{\mathrm{C}}$ being a maximum), and equal or less[34] than n_Ω, then every other $\Omega^{n\Omega}_{\tau_i}$ undergoes from the relation: $\Omega^{n\Omega}_{\tau_i} = \mathring{\Pi}^{n\Omega}_{\tau_i} \frac{\Omega^{n\Omega}_{\tau_j}}{\max_j \mathring{\Pi}^{n\Omega}_{\tau_j}}$.

At this point, the quantity W can be defined as *prescriptive verification function* or or PV which is simply the sum of the a-rational verification function Ω with the rational verification function V.

$W^n_{\tau_i}$ represents the number of (a-rational and rational) *verifications of a relevant theory* τ_i *at* $t = n$ (n included).

$$W^n_{\tau_i} = \Omega^{n\Omega}_{\tau_i} + V^n_{\tau_i} \tag{21}$$

The idea is now to sum up all the verifications of all relevant theories with value of probability $\Pi^{hy}_{x\tau_i}$ (that is, over all theories of $\mathscr{T}$) in order to get $W^n_{\mathscr{T}}$, the *weight of the probability* Π^{hy}_x, which has a leading role in achieving the prescriptive degree of certainty of future data. In fact, this *PDC* is obtained as a *w*eighted[35] sum of all hypothetical probabilities with respect to their *w*eight $W^n_{\mathscr{T}}$.

But the formalization of this idea is not so direct: for some values of hypothetical probability $W^n_{\mathscr{T}}$ can be finite, while infinitesimal for others, and their weighted sum is traditionally treated by two different formal schemes. So, Psyche devised[36] the following *nabla operator*[37] to reflect the unitary conceptual phenomenology (which is visualized by the wheels of `Figure 1`) into one single formal mechanism:

$$\nabla\left(W^n_{\tau_i}\right) = \begin{cases} W^{n\prime}_{\tau_i} & \text{where } W^n_{\tau_i} \text{ has derivative} \\ \delta_{\tau_i} W^n_{\tau_i} & \text{where } W^n_{\tau_i} \text{ is discrete} \end{cases} \tag{23}$$

[34] Actually, equal or less if there is no innate evidence.

[35] Informally, among the previous quantities the following mnemonic equation holds:

$$T \cdot U = V = W - \Omega \tag{22}$$

[36] Actually the matter is not as simple as outlined here. I have developed some tools to deal with it more properly, but I will expose them in another paper. Anyway it is a lateral technical topic that can be related mainly to issues of philosophy of mathematics. The problem arises for a mathematical function may be continuous, but without having derivative.

[37] Nabla is the Greek name of a stringed instrument similar to a harp. This operator is to "create harmony" between discrete and continuous distributions. This summary contains a simpler version of ∇.

where δ_{τ_i} stands for a Dirac delta function peaked at the value of Π_x^{hy}, for which $W_{\tau_i}^n$ is discrete[38].

By defining the *weight of the probability* Π_x^{hy} as

$$W_{\mathscr{T}}^n = W_{\mathscr{T}}^n(\Pi_x^{hy}) = \sum_{\tau_i \in \mathscr{T}} \nabla \left(W_{\tau_i}^n(\Pi_{x\tau_i}^{hy}) \right) \tag{25}$$

it's finally possible to write the expression for Π_x, the *PDC* of the next datum x:

$$\Pi_x = \frac{\int_0^1 \Pi_x^{hy} W_{\mathscr{T}}^n(\Pi_x^{hy})\, d\Pi_x^{hy}}{\int_0^1 W_{\mathscr{T}}^n(\Pi_x^{hy})\, d\Pi_x^{hy}} \tag{26}$$

The *PDC* is a probability measure function Π derived as a *w*eighted[39] sum of the probability of the future evidence, according to every relevant theory, with respect to the verification of the theories. The definition of *probability system*[40] that stands for the ordered quadruple $(\mathbf{T}, \sigma, \Sigma, \Pi)$ (whereas (σ, Σ, Π) is the usual probability space and Σ its σ-algebra) can display the fundamental key that uniquely constrains any Kolmogorovian probability measure: the choice of the set of the CMS-theories to experiment $\mathbf{T}$.

It may be illuminating to visualize the last formula as the area under the curve in the graph $W_{\mathscr{T}}^n(\Pi_x^{hy})$, which will be called *spectrum of probability*. When there are only discrete values, the spectrum of probability can take the form of a histogram, but however, it can also be continuous (with respect to Π_x^{hy}): if continuity is not strictly allowed by V, by a-rational and subjective assessments of $\Omega_{\mathscr{T}}^{n\Omega}$ this can be acceptable.

To mark in which way the probability depends only on the transparency and umpire functions through the choice of $\mathbf{T}$, the following formula can be yielded:

$$W_{\mathscr{T}}^n = \sum_{\tau_i \in \mathscr{T}} \nabla \bigoplus_{\rho \in \tau_i} (T_\rho^n \cdot U_\rho^n) \tag{27}$$

[38]If $W_{\tau_i}^n$ is discrete, the Dirac delta function has the fundamental property

$$\frac{\int_0^1 \Pi_x^{hy} \left(\sum_{\tau_i \in \mathscr{T}} \delta_{\tau_i} W_{\tau_i}^n \right) d\Pi_x^{hy}}{\int_0^1 \left(\sum_{\tau_i \in \mathscr{T}} \delta_{\tau_i} W_{\tau_i}^n \right) d\Pi_x^{hy}} = \frac{\sum_{\tau_i \in \mathscr{T}} \Pi_{x\tau_i}^{hy} \cdot W_{\tau_i}^n}{\sum_{\tau_i \in \mathscr{T}} W_{\tau_i}^n} \tag{24}$$

that is, automatically reduces from integral to sum.

[39]This *w*eight $W_{\mathscr{T}}^n$ (or its normalization) can be then thought of as a sort of *meta-probability*.

[40]Terminology suggested by Gillies' works.

From which, it may be relevant to observe that the probability of future ultimately depends only on the choice of the theories used to experiment and on all their related $\Pi^{hy}_{x\rho}$'s and $F^{n}_{x\rho}$'s. Even so, the *PDC* takes account also of the data order by means of $\mathbf{T}$: the (experimental) verifications of a particular theory weigh also its order. Therefore *PDC* can perhaps can be qualified as "absolute-frequency-and-order based".

A last observation is important before ending the paragraph: it's true that Tyche might have decided to use the wheel χ_2 for the instants $t = 4, 8, 12, \ldots$, but she might have opted also for a different theory, maybe choosing the wheels according to "how she feels like most" at every instant. Since it's not possible to contemplate all theories that Tyche can think about, then how to determine the *PDC*?

In the same way: just testing theories! Some of them will be verified more and others less. And maybe Tyche herself might discover that her a-rational choices have followed some rational theory that she wasn't even aware to follow[41].

In the Psyche's context, the debate objectivism/subjectivism for the concept of probability would reduce to discuss whether or not it is possible to contemplate *all* theories for data in order to determine one "absolute" probability not dependent on the choice of $\mathbf{T}$. Easy counterexamples prove it is not possible: examples to show that not every theory has been contemplated. On the other hand, sharing the same data and the same choice of $\mathbf{T}$ the estimate of the *p*robability Π becomes an intersubjective issue[42].

However, the verifications of a theory don't depend on the choice of $\mathbf{T}$. Is the *PV* an objective quantity then? I'd rather say that *PV* is just prescriptive: whether or not this quantity or method are mind-independent or seem mind-independent is just a orthogonal issue – what matters is what they really seem they ought to be.

And is the method for determining the *PV* objective? This is tantamount to asking whether a prescription is objective, but does it really matter: that prescription that is what we want anyway.

[41]Tyche could have some rational theories on how she does, but the only experience confirms more or less a particular theory with respect to another one. On the other hand, she might also have mistaken to compute verifications. This is just the consideration of fallibility, but also such a fallibility is "*regula*ted" by quantitative probabilistic *rule*s.

[42]*PDC* could be said to be *omnijective* as it has to work for both "objective" and subjective data.

This is in line with the more general *CMS*-method: it is "pure method" and works in a realist context and in an anti-realist context alike.

Finally, it can also be important to say that *PDC* fulfills all three Kolmogorov's axioms[43] (with countable additivity).

Confirmation corollary

If $V^n_{\tau_i}$ was the rational verification of a relevant theory *at instant* n (where n is included) it is easily generalized to the *rational verifications of a theory at instant* t (with t included) $V^t_{\mathcal{T}_i}$. This is obtained by generalizing V^n_ρ in V^t_ρ computed as if that generic instant t (t included) was the last instant of the past (that is, the superscipt of the rational verification function defines the last instant of the experimental contextual location for the considered problem and for the past the experience that hypothetically finishes after that).

$V^t_{\mathcal{T}_i}$ simply follows as

$$V^t_{\mathcal{T}_i} = \bigoplus_{\rho \in \mathcal{T}_i} V^t_\rho \tag{29}$$

Mutatis mutandis, also $\Omega^t_{\mathcal{T}_i}$ is derived in order to get

$$W^t_{\mathcal{T}_i} = \Omega^{t_\Omega}_{\mathcal{T}_i} + V^t_{\mathcal{T}_i} \tag{30}$$

At this point, the following results appear natural:

x_{t+1} *confirms* the theory $\mathcal{T}_i$, in quantity $W^{t+1}_{\mathcal{T}_i}$, when

$$W^t_{\mathcal{T}_i} < W^{t+1}_{\mathcal{T}_i} \tag{31}$$

x_{t+1} is *neutral* with respect to the confirmation of the theory $\mathcal{T}_i$ when

$$W^t_{\mathcal{T}_i} = W^{t+1}_{\mathcal{T}_i} \tag{32}$$

x_{t+1} *disconfirms* the theory $\mathcal{T}_i$ when

$$W^t_{\mathcal{T}_i} > W^{t+1}_{\mathcal{T}_i} \tag{33}$$

[43] For, if $\mathcal{E}_V$ is discrete then

$$\sum_{x\in\sigma} \Pi_x = \sum_{x\in\sigma} \frac{\sum\limits_{\tau_i\in\mathscr{T}} \Pi^{hy}_{x\tau_i} \cdot W^n_{\tau_i}}{\sum\limits_{\tau_i\in\mathscr{T}} W^n_{\tau_i}} = \frac{\sum\limits_{\tau_i\in\mathscr{T}} \sum\limits_{x\in\sigma} \Pi^{hy}_{x\tau_i} \cdot W^n_{\tau_i}}{\sum\limits_{\tau_i\in\mathscr{T}} W^n_{\tau_i}} = \frac{\sum\limits_{\tau_i\in\mathscr{T}} \left(W^n_{\tau_i} \sum\limits_{x\in\sigma} \Pi^{hy}_{x\tau_i} \right)}{\sum\limits_{\tau_i\in\mathscr{T}} W^n_{\tau_i}} = \frac{\sum\limits_{\tau_i\in\mathscr{T}} W^n_{\tau_i}}{\sum\limits_{\tau_i\in\mathscr{T}} W^n_{\tau_i}} = 1 \tag{28}$$

If $\mathcal{E}_V$ is a continuum then it can be approximated as a limit of the discrete case.
Finally, $\mathcal{E}_\Omega$ forms $W^n_{\mathscr{T}}$ (or ought to do, as *PDC* is prescriptive) in the way that $\mathcal{E}_V$ does.

and x_{t+1} *falsifies* the theory $\mathcal{T}_i$ when

$$W^t_{\mathcal{T}_i} > 0 \quad \text{and} \quad W^{t+1}_{\mathcal{T}_i} = 0 \ . \tag{34}$$

Note: if (at least) one single hypothetical datum is falsified then it falsifies its theory forever. Furthermore, one single experimental datum can falsify or confirm a whole set of theories at the same time, but, in general, this is done in different quantity.[44]

Technical note: by the chosen definitions, even if $t+1 \notin \lambda^{ex}_{\mathcal{T}_i}$ (not necessarily $t \in \lambda^{ex}_{\mathcal{T}_i}$ even if $t+1$ belongs to the past) or if $t+1 \notin \lambda^{hy}_{\mathcal{T}_i}$ then x_{t+1} is *neutral* with respect to the confirmation of the theory $\mathcal{T}_i$[45].

Finally it's important to observe that the notion of confirmation of a theory does in general not depend on the probabilities of that theory[46].

Causal corollary

The basic idea is this: if a theory is confirmed then also its hypothetical causes are confirmed.

A rule ρ is called a *causal rule with probability* $\Pi^{hy}_{x\rho}$ *for* x_t if $t \in \lambda^{hy}_{\rho}$, whereas now $\Pi^{hy}_{x\rho} \in \chi^{hy}_{\rho}$ is the hypothetical probability of datum x at instant t according to ρ.

A theory $\mathcal{T}_i$ is a *causal theory with probability* $\Pi^{hy}_{x\rho}$ *for* x_t, and indicated as $\hat{\tau}_i$, if it contains a causal rule. $\hat{\mathscr{T}}$ is the set of all causal theories.

$$V^n_{\hat{\tau}_i} = \bigoplus_{\rho \in \hat{\tau}_i} V^n_{\rho} \tag{35}$$

$$W^n_{\hat{\tau}_i} = \Omega^{n\Omega}_{\hat{\tau}_i} + V^n_{\hat{\tau}_i} \tag{36}$$

[44] These last two *experimentum-crucis*-like instances are the methodological basis to face holistic issues.

[45] Mainly for the definition of rational verifications of a rule $V^{t+1}_{\rho} = T^{t+1}_{\rho} \cdot U^{t+1}_{\rho} = T^t_{\rho} \cdot U^t_{\rho}$ which is dependent only on the instants which are prior to the minimum between $t+1$ and n (being both included).

[46] While this is the case for Bayesianism. Besides, in spite of Bayesianism, also old evidence can confirm, and the probability of a theory is mathematically undefined unless there is some evidence to suport it, and many many other qualities.

and[47]

$$W^n_{\hat{\mathscr{T}}} = W^n_{\hat{\mathscr{T}}}(\Pi^{hy}_x) = \sum_{\hat{\tau}_j \in \hat{\mathscr{T}}} \nabla \left(W^n_{\hat{\tau}_j}(\Pi^{hy}_{x\hat{\tau}_j}) \right) \tag{37}$$

The hypothetical cause of the causal theory, $\mathcal{C}^{hy}_{\hat{\tau}_i}$, can be said *to have PDC not less than* $\xi^t_{\hat{\tau}_i}$ if

$$\frac{W^n_{\hat{\tau}_i}}{\int_0^1 W^n_{\hat{\mathscr{T}}}(\Pi^{hy}_x)\, d\Pi^{hy}_x} = \xi^t_{\hat{\tau}_i} \tag{38}$$

The hypothetical cause of the causal theory, $\mathcal{C}^{hy}_{\hat{\tau}_i}$, is said *to have PDC* $\Pi^t_{\hat{\tau}_i}$ if $\Pi^t_{\hat{\tau}_i}$ is the sum of the $\xi^t_{\hat{\tau}_i}$'s over all the theories such that they have the same hypothetical cause of the causal rule for x_t (cases of two or more theories with the same cause).

If $\Pi^t_{\hat{\tau}_i} = 1$ then $\mathcal{C}^{hy}_{\hat{\tau}_i}$ is said to be the *cause of* x_n[48].

In other words, $\Pi^t_{\hat{\tau}_i}$ is the *PDC*[49] *of the hypothetical cause* $\mathcal{C}^{hy}_{\hat{\tau}_i}$ that can be seen as the *proportion played by the concause* $\mathcal{C}^{hy}_{\hat{\tau}_i}$ for the datum x_t. The name "concause" is because all theories (which are relevant and not falsified) must be considered at the same time *to explain* the datum x_t as long as there is reason to neglect some concauses (or theories) and this reasonableness is merely determined by the computation of Π.

Note: the definition of causal rule does *not* require t to belong to λ^{ex}_ρ only. This might look inadequate, for example, in the case where the next a has a cause to be a (with $\Pi^n_{\hat{\tau}_i} = 1$) and the possible next experience of b seems to counter accepting a to as the cause of b. So, this could suggest to limit the computation of the *PDC* of hypothetical causes to only the past λ^{ex}_ρ: only for causes of past data. But, on the other hand, also

[47]It may be useful to remember the differences among the verifications of the three different types of theories treated here:

$V^n_{\mathbf{T}}$ equals either 0 or the number of rational verifications at instant n (n included) of all the theories $\mathcal{T}_i$.

$V^n_{\mathscr{T}}$ equals either 0 or the number of rational verifications at instant n (n included) of all the theories $\mathcal{T}_i$ such that they have a rule for which $n+1 \in \lambda^{hy}_{\mathcal{T}_i}$, where they are assigned a wheel with Π^{hy}_x.

$V^n_{\hat{\mathscr{T}}}$ equals either 0 or the number of rational verifications at instant n (n included) of all the causal theories $\mathcal{T}_i$ for x_t such that they have a rule for which $t \in \lambda^{hy}_{\mathcal{T}_i}$, where they are assigned a wheel with Π^{hy}_x.

[48]This concept of cause is different from the one of hypothetical cause, in the analogous way in which the concept of probability (*PDC*) is different from the one of hypothetical probability.

[49]Clearly it is a Kolmogorovian probability:

$$\sum_{\text{all } \hat{\tau}_i} \Pi^n_{\hat{\tau}_i} = \frac{\int_0^1 W^n_{\hat{\mathscr{T}}}(\Pi^{hy}_x)\, d\Pi^{hy}_x}{\int_0^1 W^n_{\hat{\mathscr{T}}}(\Pi^{hy}_x)\, d\Pi^{hy}_x} = 1 \tag{39}$$

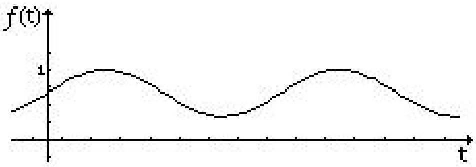
a cause which ha produced a past datum with $\Pi^{t}_{\hat{\tau}_i} = 1$ (established by analysis of the precedent experience) can be "falsified" by future data.

Corollary of the *PDC* of theories

Here follows an example of an CMS-theory: let $\mathcal{T}_{123}$ be the off-line omnipresent theory, decided by Tyche[50], so that the hypothetical probability of datum a goes as such

$$f(t) = \frac{1}{3}\sin t + \frac{2}{3} \tag{40}$$

which has the graph of `Figure 5`

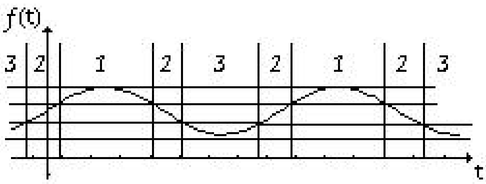


`Figure 5`

and the hypothetical probability of datum b goes as follows $1 - f(t)$.

The problem is now: how to determine the *PDC* of the theory?

Roughly speaking, the idea is now to chop up the sine into several slices in order to get several sets of intervals which have nearly the same hypothetical random variable. Therewith several rules have been obtained, like the illustrative rules with $\rho = 1, 2, 3$ of the following `Figure 6`.



`Figure 6`

Supposing that n instants are reckoned until the present and that the problem is to compute the probability of theory $\mathcal{T}_{123}$ (which is also relevant), then the solution is natural: $W^{n}_{\tau_{123}}$ expresses the present verifications of τ_{123}. To get the *PDC* of τ_{123} then $W^{n}_{\tau_{123}}$ must be divided by the sum of the verifications of all present relevant theories.

[50] Otherwise, another cause could be a quantum mechanical law, from which this formula is derived.

Namely

$$\frac{W^n_{\tau_i}}{\int\limits_0^1 W^n_{\mathscr{T}}\,(\Pi^{hy}_x)\,d\Pi^{hy}_x} = \Pi^n_{\tau_i} \tag{41}$$

is the *PDC*[51] *of the theory τ_i for datum x at instant n* (that is, the present probability of the theory τ_i for the next future datum[52]).

Therefore, in a similar way to what happens with the *PDC* of data, the *PDC* of a theory[53] for some data always depends on its intrinsic verifications, but always also in the comparison with other known experiment*ed* theories.

However, the *PDC* of data according to some hypothetical theory (which is a probability of data) is a different concept from the one of the *PDC* of a theory for those same data (which is a probability of data): the former is a weighted sum and the latter is a ratio[54]. Nevertheless, they are linked by the following relation:

$$\Pi_x = \int\limits_0^1 \Pi^{hy}_x \left(\sum_{\tau_i \in \mathscr{T}} \nabla \left(\Pi^n_{\tau_i}\,(\Pi^{hy}_{x\tau_i}) \right) \right) d\Pi^{hy}_x \tag{43}$$

Examples: *A I* ... *A*rtificial *I*ntelligence?

Two easy practical strictly-deterministic[55] examples are shown now: the strings A and I. Elaboration of more complex and strictly-probabilistic cases has no formal differences.

String A can be named "(falling) *a*pple", in fact there is a perfect analogy with the

[51] Which clearly fulfills Kolmogorov's axioms for

$$\sum_{\text{all } \tau_i} \Pi^n_{\tau_i} = \frac{\int\limits_0^1 W^n_{\mathscr{T}}\,(\Pi^{hy}_x)\,d\Pi^{hy}_x}{\int\limits_0^1 W^n_{\mathscr{T}}\,(\Pi^{hy}_x)\,d\Pi^{hy}_x} = 1 \tag{42}$$

[52] If, instead of the next future datum, the interest is for another instant, then the more general *PDC* of the theory τ_i for the datum x_t is given by $\xi^t_{\hat{\tau}_i}$: identical to the *PDC* of the causal theory (if $t = n$ then $\xi^t_{\tau_i} = \Pi^n_{\tau_i}$.)

[53] Like $\mathcal{T}_{123}$, or the law of universal gravitation, or like the whole quantum mechanics.

[54] While in Bayesianism's view they'd receive the same treatment.

[55] By "strictly-deterministic" and "strictly-probabilistic' I refer to the meaning which is often used in the philosophy of physics: associated an intrinsic probability 1 or 1 and intermediate respectively. 'Hard determinism' is commonly used to mean that free will is an illusion.

"time-squared law" for freely falling bodies. It is defined as

$$A = \{x_t\} \quad \text{where} \quad x_t = \begin{cases} a, & t = k^2 \text{ with } k \in \mathbb{N} \\ b, & t \neq k^2 \text{ with } k \in \mathbb{N} \end{cases} \tag{44}$$

and the instants of a's are numerically equal to the measure of distance of an apple from the initial position[56]. If now the original Psyche's white color is indicated by a white square $\square$ (instead of an a) and the $b$lack color is indicated by a black square $\blacksquare$ then the display of the string visually reminds to the stroboscopic photograph of a falling apple:

$$A = \square, \blacksquare, \blacksquare, \square, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \square, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \square, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \ldots \tag{45}$$

String I can be named "$i$ntermittence", in fact there is a perfect analogy with a Tyche's intermittent illumination of Hypoland. It is defined as

$$I = \{x_t\} \quad \text{where} \quad x_t = \begin{cases} a, & t = 2k - 1 \text{ with } k \in \mathbb{N} \\ b, & t = 2k \text{ with } k \in \mathbb{N} \end{cases} \tag{46}$$

and can be displayed as

$$I = \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \ldots \tag{47}$$

Strings A and I are representations of two different off-line omnipresent relevant theories (made up by two CMS-rules each corresponding to the degenerate random variable for the white and for the black). The work on the latter string is shown first for it's easier.

As the past evidence is finite, it's suitable to define a I_ϕ as the "$f$inite part" of I such that ϕ corresponds to the number of $f$avorably-verified cases for $\square$ and the lenght of I_ϕ is of one instant shorter than the instant where the square $\square$ of number $\phi + 1$ is located:

$I_1 = \square, \blacksquare$

$I_2 = \square, \blacksquare, \square, \blacksquare$

$I_3 = \square, \blacksquare, \square, \blacksquare, \square, \blacksquare$

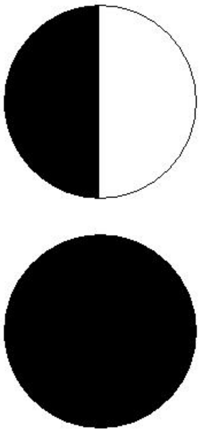
$\vdots$

At this point the problem is the following: supposing to have experienced only an I_ϕ, how to assess whether I_ϕ is the strictly-deterministic product of the Tyche's planned

[56] A has also the curious aesthetic property that the number of b's between the a's is equal to $2k$: 2,4,6,8,...

intermittent illumination or whether it's the random product of a Tyche's tossing of a perfect coin? Then, how to assess the prescriptive degree of certainty of a next $\square$ by means of the previous conclusion?

Painting the wheels χ_7 and χ_8 of `Figure 7` it is possible to reduce to the defined Psy-



`Figure 7`

che's formalization. In fact, the theory *i*ntermittence is the set $\tau_I = \left\{ \left(\mathcal{C}_4^{hy}, \lambda_4^{hy}, \chi_4^{hy} \right), \left(\mathcal{C}_8^{hy}, \lambda_8^{hy}, \chi_8^{hy} \right) \right\}$ and the theory *c*oin is the set $\tau_C = \left\{ \left(\mathcal{C}_7^{hy}, \lambda_7^{hy}, \chi_7^{hy} \right) \right\}$ whereas, clearly, χ_4^{hy}, χ_7^{hy}, χ_8^{hy} are wheels χ_4,χ_7,χ_8, $\mathcal{C}_4^{hy} = \mathcal{C}_8^{hy}$ is Tyche's intermitting illumination, $\lambda_4^{hy} = \{t = 2k - 1 \text{ with } k \in \mathbb{N}\}$, $\lambda_8^{hy} = \{t = 2k \text{ with } k \in \mathbb{N}\}$, $\mathcal{C}_7^{hy}$ is Tyche's random illumination from a perfect-coin tossing[57], $\lambda_7^{hy} = \mathbb{N}$.

Therefore

$$V_{\tau_I}^n = V_{\tau_I}^n \left(\Pi_{\square \tau_I}^{hy} = 1 \right) = V_4^n + V_8^n = 2 \cdot T_4^n \cdot \eta_4^n \cdot \theta_4^n = n = 2\phi \tag{48}$$

while

$$V_{\tau_C}^n = V_{\tau_C}^n \left(\Pi_{\square \tau_I}^{hy} = \frac{1}{2} \right) = V_7^n = T_7^n \cdot \eta_7^n \cdot \theta_7^n = 2\phi \cdot \frac{(2\phi)!}{(\phi!)^2} \cdot \left(\frac{1}{2} \right)^{2\phi} \tag{49}$$

and so the first question of the problem has already received answer: indeed, $V_{\tau_I}^n$ and $V_{\tau_C}^n$ are the verifications of theory I and C respectively. Then, the PDC of $\square$ at $t = 2\phi - 1 = n + 1$ will be simply[58]

$$\Pi_{\square} = \frac{1 \cdot V_{\tau_I}^n + \frac{1}{2} \cdot V_{\tau_C}^n}{V_{\tau_I}^n + V_{\tau_C}^n} \tag{50}$$

[57] Which is obviously identical to the spinning of wheel χ_7.

[58] In the case of having considered the only two theories I and C.

which, as expected, approaches 1 as ϕ approaches ∞.

Analogously to what just done, A_ϕ will be a "*f*inite part" of A such that ϕ corresponds to the number of *f*avorably-verified cases for $\square$ and the lenght of I_ϕ is of one instant shorter than the instant where the square $\square$ of number $\phi+1$ is located:

$A_1 = \square, \blacksquare, \blacksquare$

$A_2 = \square, \blacksquare, \blacksquare, \square, \blacksquare, \blacksquare, \blacksquare, \blacksquare$

$A_3 = \square, \blacksquare, \blacksquare, \square, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \square, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare$

$\vdots$

Still the problem is: how to assess whether A_ϕ is the strictly-deterministic product of the Tyche's planned intermittent illumination or whether it's the random product of a Tyche's tossing of an imperfect coin? Then, how to assess the prescriptive degree of certainty of a next $\square$ by means of the previous conclusion?

The theory *a*pple reduces to the set $\tau_A = \left\{ \left(\mathcal{C}^{hy}_{40}, \lambda^{hy}_{40}, \chi^{hy}_{40} \right), \left(\mathcal{C}^{hy}_{80}, \lambda^{hy}_{80}, \chi^{hy}_{80} \right) \right\}$ and the theory *imperfect* *c*oin[59] is the set $\tau_{\not{C}} = \left\{ \left(\mathcal{C}^{hy}_{90}, \lambda^{hy}_{90}, \chi^{hy}_{90} \right) \right\}$ whereas, clearly, χ^{hy}_{40} and χ^{hy}_{80} are wheels χ_4 and χ_8, $\mathcal{C}^{hy}_{40} = \mathcal{C}^{hy}_{80}$ is Tyche's illumination according to the integer squared, $\lambda^{hy}_{40} = \mathbb{N} \backslash \lambda^{hy}_{40}$, $\lambda^{hy}_{80} = \{t = 2k \text{ with } k \in \mathbb{N}\}$, $\mathcal{C}^{hy}_{90}$ is Tyche's random illumination from an imperfect-coin tossing[60] and $\lambda^{hy}_{80} = \mathbb{N}$.

Therefore

$$V^n_{\tau_A} = V^n_{\tau_A} \left(\Pi^{hy}_{\square\tau_A} = 1 \right) = V^n_{40} + V^n_{80} = n = (\phi+1)^2 - 1 = \phi^2 + 2\phi \tag{51}$$

while

$$\begin{aligned} V^n_{\tau_{\not{C}}} = V^n_{\tau_{\not{C}}} \left(\Pi^{hy}_{\square\tau_{\not{C}}} = \tfrac{1}{\phi+2} \right) = V^n_{90} = T^n_{90} \cdot \eta^n_{90} \cdot \theta^n_{90} = \\ = \left((\phi+1)^2 - 1 \right) \cdot \left(\tfrac{(\phi^2+2\phi)!}{\phi!(\phi^2+\phi)!} \right) \cdot \left(\left(\tfrac{1}{\phi+2} \right)^{\phi} \left(\tfrac{\phi^2+\phi}{\phi^2+2\phi} \right)^{\phi^2+\phi} \right) \end{aligned} \tag{52}$$

and $V^n_{\tau_I}$ and $V^n_{\tau_{\not{C}}}$ are the verifications of theory A and $\not{C}$ respectively. Then, the *PDC* of $\square$ at $t = \phi^2 = n+1$ will be simply[61]

$$\Pi_\square = \frac{1 \cdot V^n_{\tau_A} + \frac{1}{\phi+2} \cdot V^n_{\tau_{\not{C}}}}{V^n_{\tau_A} + V^n_{\tau_{\not{C}}}} \tag{53}$$

[59] $\not{C}$ reads "C slash".

[60] Which is obviously identical to the spinning of a wheel 90, made of the only color back and white. This wheel is different at each problem from the considered A_ϕ for having a $\Pi^{hy}_{\square 90} = \Pi^{ex}_{\square 90} = \frac{1}{\phi+2}$ at every A_ϕ.

[61] In the case of having considered the only two theories A and $\not{C}$.

For both A_ϕ and I_ϕ, as ϕ approaches ∞, PDC of $\square$ approaches 1. In other words, for the two considered examples, the longer the string gets, the more the algorithm of PDC "understands" that the strictly-deterministic theories are more probable[62] for the data considered. The position where the apple will be at next instant is predicted automatically (and without having talked much about physics).

Therefore PDC works also as a *machine learning algorithm.*

It can be relevant[63] to observe that, in comparison with strictly-probabilistic theories, not for every considered strictly-deterministic unfalsified relevant theory its PDC approaches 1: for example, using the "subtheories" given by the only rules with ρ equals 4 and 40 (τ_4 and τ_{40}) then:

$$\Pi^n_{\tau_4} = \frac{V^n_{\tau_4}}{V^n_{\tau_4} + V^n_{\tau_C}} \xrightarrow{n\to\infty} 1 \tag{54}$$

but

$$\Pi^n_{\tau_{40}} = \frac{V^n_{\tau_{40}}}{V^n_{\tau_{40}} + V^n_{\tau_{\not C}}} \xrightarrow{n\to\infty} 0 \ . \tag{55}$$

However, any strictly-deterministic unfalsified theory is always more verified than any strictly-probabilistic one if applied to the same number of data. That's why it is usually *inter-subjectively* agreed that

$$\Pi\left(\square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \square, \blacksquare, \ldots, \square, \blacksquare, \square, \blacksquare; \square\right) = 1 \tag{56}$$

without having discussed[64] about the considered choice for $\mathbf{T}$.

Finally, a last example which can be worked out only by a sentence is the Psyche's counterpart for the Goodman's paradox of grue emeralds: 'The world is *blite*'. Blite is a hypothetical color defined as *bl*ack in the whole past experience, but permanently wh*ite* after some future instant. It can be represented by the string

$$B = \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \ldots, \blacksquare, \blacksquare, \blacksquare, \blacksquare, \blacksquare; \square, \square, \square, \square, \square, \square, \square, \square, \square \ldots \tag{57}$$

[62]With a PDC $\Pi^n_{\tau_A} = \frac{V^n_{\tau_A}}{V^n_{\tau_A} + V^n_{\tau_{\not C}}}$ or $\Pi^n_{\tau_I} = \frac{V^n_{\tau_I}}{V^n_{\tau_I} + V^n_{\tau_C}}$.

[63]Although a bit more technical.

[64]But with the obvious condition of having contemplated the obvious strictly-deterministic theory. It's very important also that the "..." forces the data fitting to a single strictly-deterministic unfalsified theory.

Even though the theory ‘The world is black’ is always verified (and has *PDC* equal to 1), the theory ‘The world is blite’ is never verified since its rule corresponding to the white wheel hasn’t had any experimental verification yet (and so the o-plus operator sets the verifications of the “entire” theory equal to zero).